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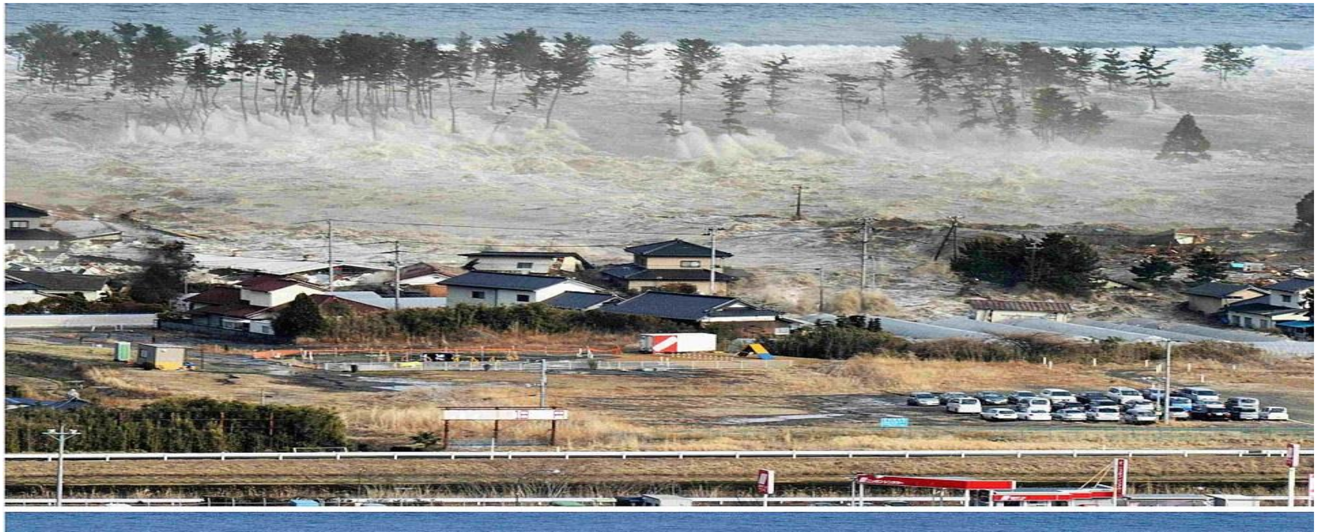
Periodic Water Waves through an Aquatic Forest

Philip L.-F. Liu[1,2], Che-Wei Chang[1], Chiang C. Mei[3], Pedro Lomónaco[4,5], Francisco L. Martín[4] and Maria Maza[4]

[1]School of Civil and Environmental Engineering, Cornell University, USA
[2]Institute of Oceanic and Hydrological Sciences, National Central University, Taiwan
[3]Department of Civil and Environmental Engineering, MIT, USA
[4]Instituto de Hidraulica Ambiental, Universidad de Cantabria, Spain
[5]O.H. Hinsdale Wave Research Laboratory, Oregon State University, USA

Philip L.-F. Liu
Class of 1912 Professor in Engineering
Director, School of Civil and Environmental Engineering
Cornell University

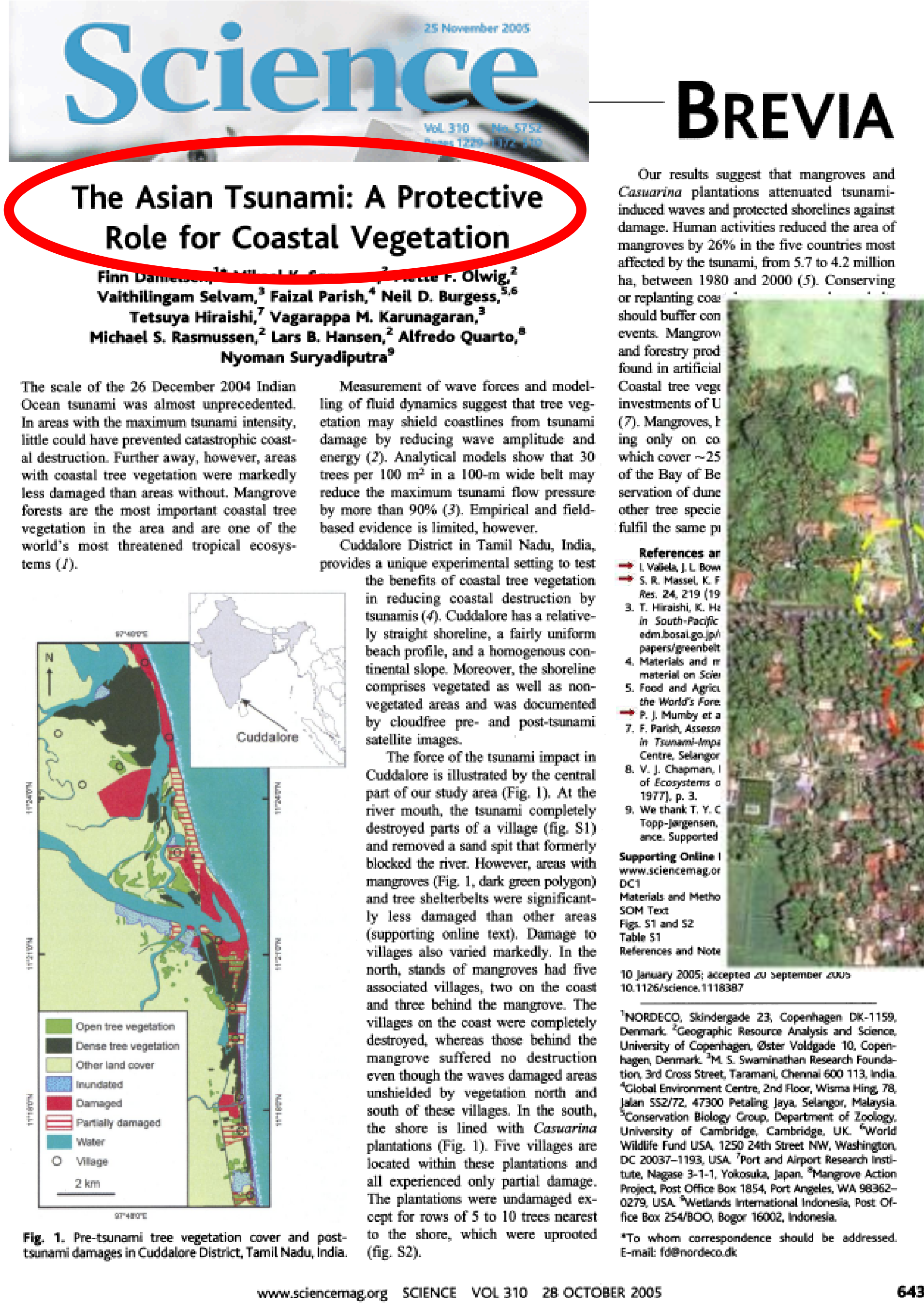


Mei, C. C., Chan, I.-C., Liu, P. L.-F., Huang, Z., and Zhang, W. Long waves through emergent coastal vegetation. Journal of Fluid Mechanics , 687:461491, 2011.

Mei, C. C., Chan, I.-C., and Liu, P. L.-F. Waves of intermediate length through an array of vertical cylinders. Environmental Fluid Mechanics , 14:235261, 2014.

Liu, P. L-F., Chang, C-W., Mei, C.C., Lomónaco, P., Martín, F. L., and Maza, M. Periodic water waves through an aquatic forest. Coastal Engineering, in press.

Motivation



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BREVIA

The Asian Tsunami: A Protective Role for Coastal Vegetation

Finn [illegible],[1*] [illegible] F. Olwig,[2]
Vaithilingam Selvam,[3] Faizal Parish,[4] Neil D. Burgess,[5,6]
Tetsuya Hiraishi,[7] Vagarappa M. Karunagaran,[3]
Michael S. Rasmussen,[2] Lars B. Hansen,[2] Alfredo Quarto,[8]
Nyoman Suryadiputra[9]

The scale of the 26 December 2004 Indian Ocean tsunami was almost unprecedented. In areas with the maximum tsunami intensity, little could have prevented catastrophic coastal destruction. Further away, however, areas with coastal tree vegetation were markedly less damaged than areas without. Mangrove forests are the most important coastal tree vegetation in the area and are one of the world's most threatened tropical ecosystems (*1*).

Measurement of wave forces and modelling of fluid dynamics suggest that tree vegetation may shield coastlines from tsunami damage by reducing wave amplitude and energy (*2*). Analytical models show that 30 trees per 100 m² in a 100-m wide belt may reduce the maximum tsunami flow pressure by more than 90% (*3*). Empirical and field-based evidence is limited, however.

Cuddalore District in Tamil Nadu, India, provides a unique experimental setting to test the benefits of coastal tree vegetation in reducing coastal destruction by tsunamis (*4*). Cuddalore has a relatively straight shoreline, a fairly uniform beach profile, and a homogenous continental slope. Moreover, the shoreline comprises vegetated as well as non-vegetated areas and was documented by cloudfree pre- and post-tsunami satellite images.

The force of the tsunami impact in Cuddalore is illustrated by the central part of our study area (Fig. 1). At the river mouth, the tsunami completely destroyed parts of a village (fig. S1) and removed a sand spit that formerly blocked the river. However, areas with mangroves (Fig. 1, dark green polygon) and tree shelterbelts were significantly less damaged than other areas (supporting online text). Damage to villages also varied markedly. In the north, stands of mangroves had five associated villages, two on the coast and three behind the mangrove. The villages on the coast were completely destroyed, whereas those behind the mangrove suffered no destruction even though the waves damaged areas unshielded by vegetation north and south of these villages. In the south, the shore is lined with *Casuarina* plantations (Fig. 1). Five villages are located within these plantations and all experienced only partial damage. The plantations were undamaged except for rows of 5 to 10 trees nearest to the shore, which were uprooted (fig. S2).

Our results suggest that mangroves and *Casuarina* plantations attenuated tsunami-induced waves and protected shorelines against damage. Human activities reduced the area of mangroves by 26% in the five countries most affected by the tsunami, from 5.7 to 4.2 million ha, between 1980 and 2000 (*5*). Conserving or replanting coa[…] should buffer con[…] events. Mangrov[…] and forestry prod[…] found in artificial[…] Coastal tree vege[…] investments of U[…] (*7*). Mangroves, b[…] ing only on co[…] which cover ~25[…] of the Bay of Be[…] servation of dune[…] other tree specie[…] fulfil the same p[…]

Fig. 1. Pre-tsunami tree vegetation cover and post-tsunami damages in Cuddalore District, Tamil Nadu, India.

References an[…]
1. I. Valiela, J. L. Bow[…]
2. S. R. Massel, K. F[…] *Res.* 24, 219 (19[…]
3. T. Hiraishi, K. Ha[…] in *South-Pacific*[…] edm.bosai.go.jp/[…] papers/greenbelt[…]
4. Materials and m[…] material on *Scie*[…]
5. Food and Agricu[…] *the World's Fore*[…]
6. P. J. Mumby *et a*[…]
7. F. Parish, *Assessm*[…] *in Tsunami-Impa*[…] Centre, Selangor[…]
8. V. J. Chapman, [...] of *Ecosystems o*[…] 1977), p. 3.
9. We thank T. Y. C[…] Topp-Jørgensen, […] ance. Supported[…]

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www.sciencemag.or[…]
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SOM Text
Figs. S1 and S2
Table S1
References and Note[…]

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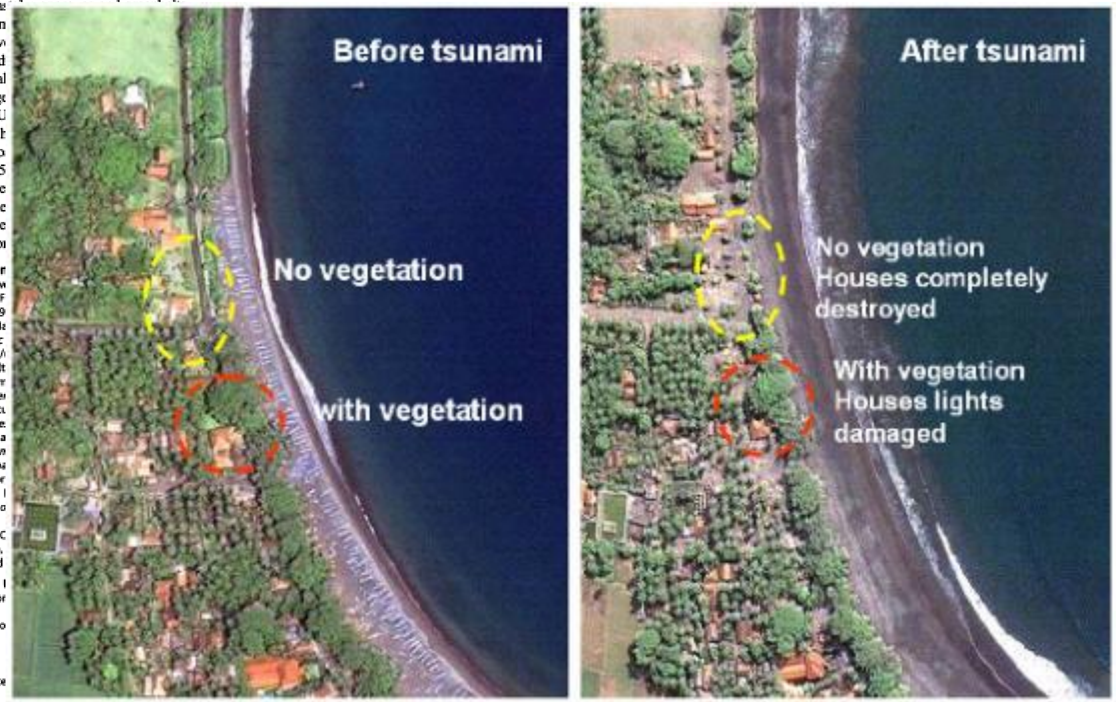
10.1126/science.1118387

[1]NORDECO, Skindergade 23, Copenhagen DK-1159, Denmark. [2]Geographic Resource Analysis and Science, University of Copenhagen, Øster Voldgade 10, Copenhagen, Denmark. [3]M. S. Swaminathan Research Foundation, 3rd Cross Street, Taramani, Chennai 600 113, India. [4]Global Environment Centre, 2nd Floor, Wisma Hing, 78, Jalan SS2/72, 47300 Petaling Jaya, Selangor, Malaysia. [5]Conservation Biology Group, Department of Zoology, University of Cambridge, Cambridge, UK. [6]World Wildlife Fund USA, 1250 24th Street NW, Washington, DC 20037–1193, USA. [7]Port and Airport Research Institute, Nagase 3-1-1, Yokosuka, Japan. [8]Mangrove Action Project, Post Office Box 1854, Port Angeles, WA 98362–0279, USA. [9]Wetlands International Indonesia, Post Office Box 254/BOO, Bogor 16002, Indonesia.

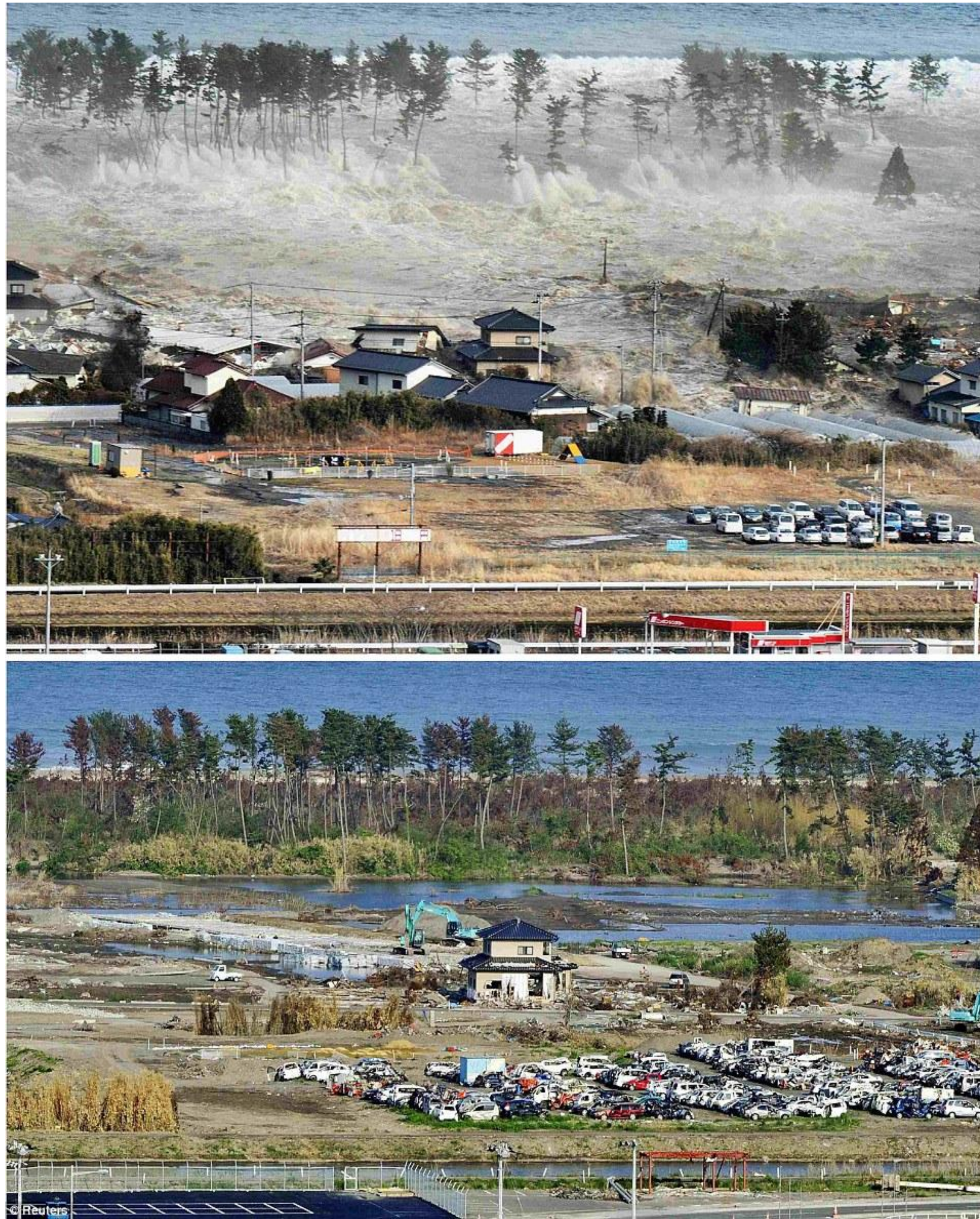
*To whom correspondence should be addressed. E-mail: fd@nordeco.dk

2006 West Java Tsunami



Source: *Coastal protection in the aftermath of the Indian Ocean tsunami*, UN report (2007).

2011 Japan Tohoku Tsunami



Natori, Miyagi Prefecture

(Photographer: Koichiro)

Motivation and Objective

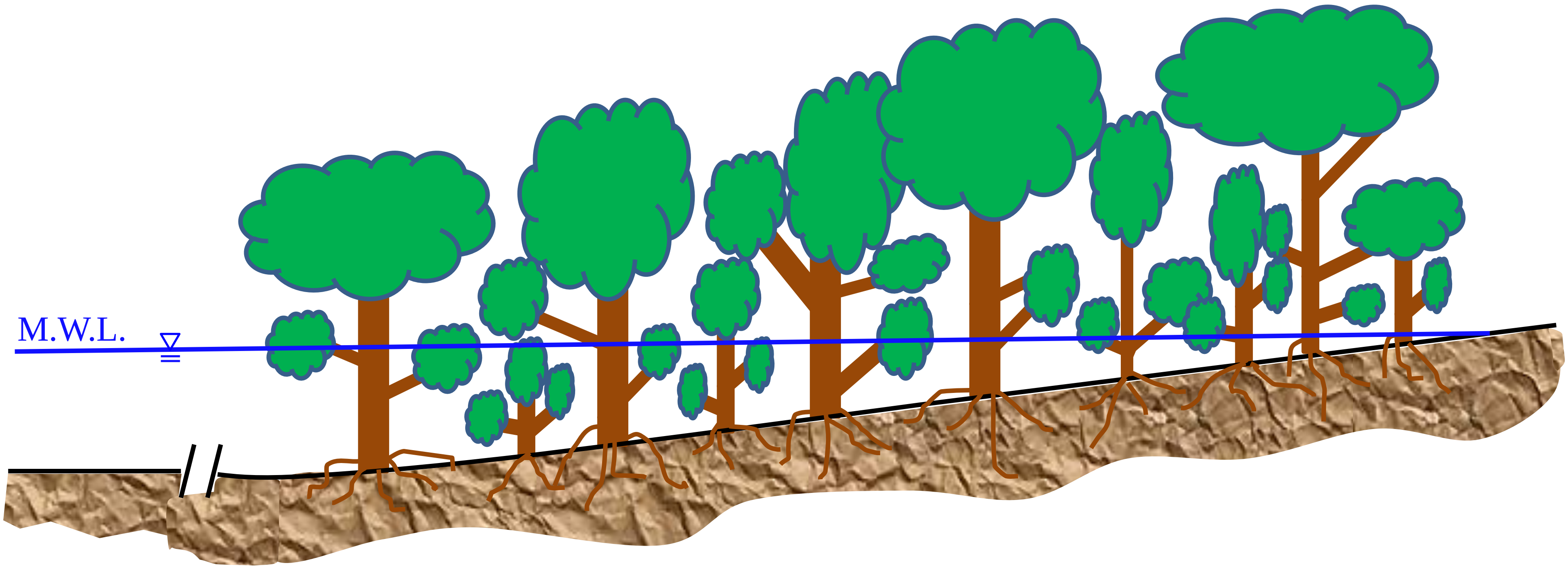
- **Motivation: *Green-belt protection***

Coastal forests as natural barriers against ocean waves

  - **Wave reflection and wave attenuation by the coastal trees**
  - **Tsunamis, storm surges, tides, daily coastal waves: limited protection!**
    - **Focus on dense coastal forests: trees that will stand!**
  - **Coastal ecosystem: dispersion coefficient, etc.**

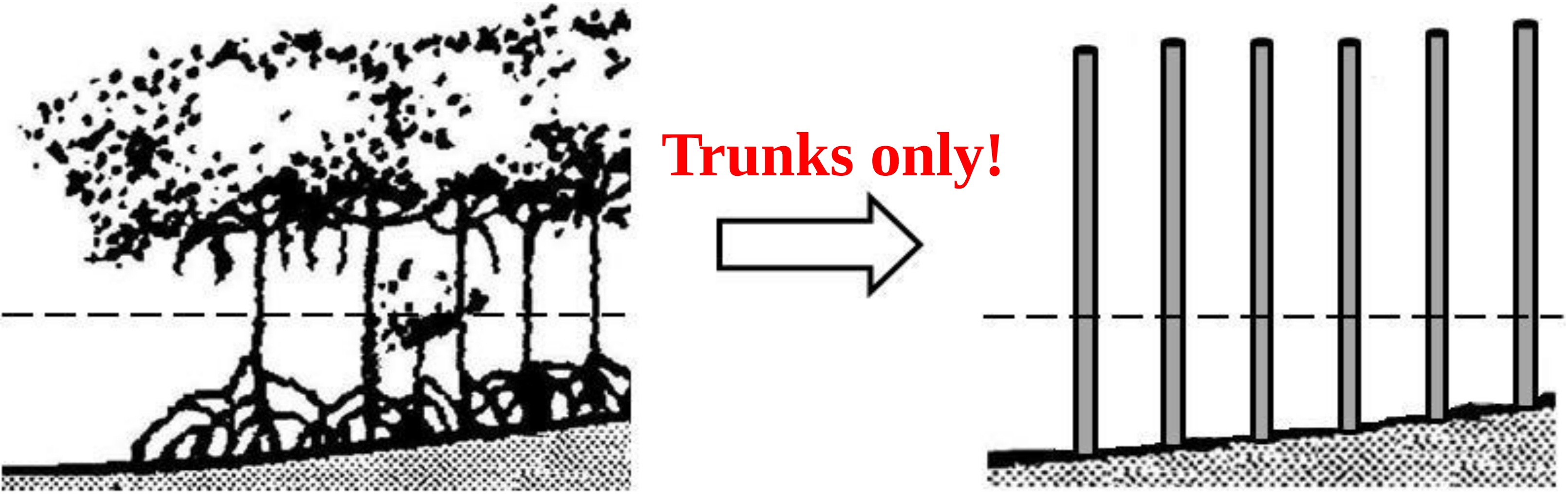
- **Objective: To develop a theoretical model for surface waves through emergent trees**
  - **Emphasize on the propagation and dissipation processes**
  - **Evaluate the effectiveness of the coastal forests**

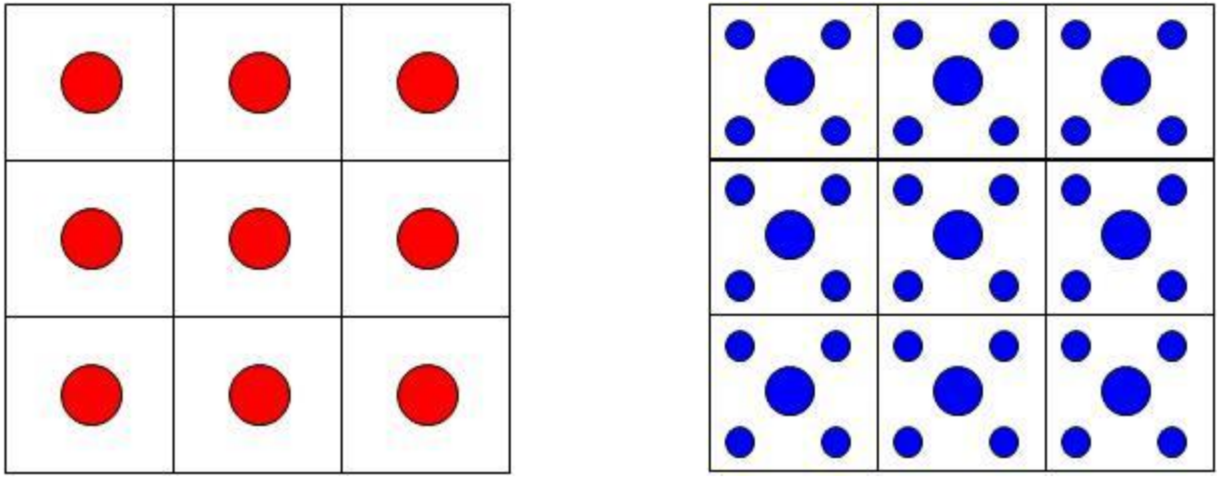
How to describe these coastal trees?



- Distribution of trees?
- Tree trunks? roots? leafs? branches?
- Soil types?
- Trunks: rigid or deformable?
- Emergent or submerged?

An idealized model: rigid cylinders





Horizontal periodicity (top view): cell concept

Mathematical models & simplifications

Basic assumptions

- Slowly varying topography
- Intermediate depth: $kh = \mathcal{O}(1) \Rightarrow$ 3D problem
- Reynolds averaged equations
- Linearized equations: small-amplitude waves
- Eddy viscosity model for wave-forest turbulence

Additional conditions

- Multiple length scales: d, ℓ, h, L
- $d, \ell \ll L \Rightarrow \epsilon = \ell/L = \mathcal{O}(0.1 \text{ m})/\mathcal{O}(100 \text{ m}) \ll 1$
- $\ell \sim h$ or $\ell \ll h$

Intermediate depth

$$kh = \mathcal{O}(1), \quad \ell \ll h$$

Two-scale analysis

- **Series expansions:** $\epsilon = \ell / L \ll 1$

$$\mathcal{F} = \mathcal{F}^{(0)} + \epsilon \mathcal{F}^{(1)} + \epsilon^2 \mathcal{F}^{(2)} + \cdots , \quad \mathcal{F} = u_i, w, p$$

$$\mathcal{F} = \mathcal{F}\left(x_k, X_k, Z, t\right), \quad k = 1, 2$$

x_k: micro scale (tree spacing ℓ)

X_k, Z: macro scales (wavelength L; depth h)

- **Homogenization: two distinct scales**
  - **Horizontal periodicity over wavelength-scale**
  - **Macro problem: leading-order equations governing wave motions with effects of rigid cylinders**
  - **Cell (micro-scale) problem: determine the effective parameters due to the presence of a costal forest**

Leading-order solutions

- **Periodic waves**

$$F(\vec{x}, \vec{X}, Z, t) = \Re\left\{\widetilde{F}(\vec{x}, \vec{X}, Z)e^{-\mathrm{i}t}\right\}$$

- **Solution forms**

$$\widetilde{u}_i^{(0)} = -\widetilde{K}_{ij}(\vec{x})\frac{\partial \widetilde{p}^{(0)}}{\partial X_j}, \quad \widetilde{w}^{(0)} = -\widetilde{W}(\vec{x})\frac{\partial \widetilde{p}^{(0)}}{\partial Z}, \quad \widetilde{p}^{(1)} = -\widetilde{A}_j(\vec{x})\frac{\partial \widetilde{p}^{(0)}}{\partial X_j}$$

- **Macro equations: wavelength-scale**

Governing equations for: $\langle \widetilde{u}_i^{(0)} \rangle$, $\langle \widetilde{w}^{(0)} \rangle$, $\widetilde{p}^{(0)}$

$\langle f \rangle = \dfrac{1}{\Omega}\displaystyle\int_{\Omega_f} f dx_1 dx_2$: **averaging over a unite cell** Ω

- **Micro (cell) equations: tree-spacing scale**

Governing equations for: $\widetilde{K}_{ij}(\vec{x})$, $\widetilde{W}(\vec{x})$

Homogenization: macro equations

$$\frac{\partial \langle \widetilde{u}_i^0 \rangle}{\partial X_i} + \frac{\partial \langle \widetilde{w}^0 \rangle}{\partial Z} = 0$$

Eddy viscosity --- to be discussed later

$$-\mathrm{i}\langle \widetilde{u}_i^0 \rangle = -n\frac{\partial \widetilde{p}^0}{\partial X_i} - \left[\oint_{S_B} ds \left(-\widetilde{A}_k \delta_{ij} + \sigma \left(\frac{\partial \widetilde{K}_{ik}}{\partial x_j} + \frac{\partial \widetilde{K}_{jk}}{\partial x_i} \right) \right) n_j \right] \frac{\partial \widetilde{p}^0}{\partial X_k}$$

$$-\mathrm{i}\langle \widetilde{w}^0 \rangle = -n\frac{\partial \widetilde{p}^0}{\partial Z} - \left[\sigma \oint_{S_B} \frac{\partial \widetilde{W}}{\partial x_j} n_j \, ds \right] \frac{\partial \widetilde{p}^0}{\partial Z}$$

- $\langle f \rangle = \frac{1}{\Omega} \int_{\Omega_f} f \, dx_1 dx_2$: **cell average;** n: porosity
- $\left(\widetilde{K}_{ij}, \widetilde{A}_j, \widetilde{W} \right)$ **to be obtained from the cell problem**
- **No forests:** $n = 1$ **and** $[\cdots\cdots] = 0$

  $\Rightarrow$ **reduce to the common wave equations**

Homogenization: cell problem

$$\widetilde{u}_i^0 = -\widetilde{K}_{ij}(\vec{x})\frac{\partial \widetilde{p}^0}{\partial X_j}, \quad \widetilde{p}^1 = -\widetilde{A}_j(\vec{x})\frac{\partial \widetilde{p}^0}{\partial X_j}, \quad \widetilde{w}^0 = -\widetilde{W}(\vec{x})\frac{\partial \widetilde{p}^0}{\partial Z}$$

- **Horizontal directions**

$$\frac{\partial \widetilde{K}_{ij}}{\partial x_j} = 0, \quad \vec{x} \in \Omega, \quad -\mathrm{i}\widetilde{K}_{ij} = \delta_{ij} - \frac{\partial \widetilde{A}_j}{\partial x_i} + \sigma \frac{\partial^2 \widetilde{K}_{ij}}{\partial x_k \partial x_k}$$

To be discussed later

- **Vertical direction**

$$-\mathrm{i}\widetilde{W} = 1 + \sigma \frac{\partial^2 \widetilde{W}}{\partial x_j \partial x_j}, \quad \vec{x} \in \Omega$$

- **Boundary conditions**

$$\text{Periodicity; } \widetilde{K}_{ij} = \widetilde{W} = 0 \text{ on } S_B; \ \langle \widetilde{A}_j \rangle = 0$$

Solution procedure

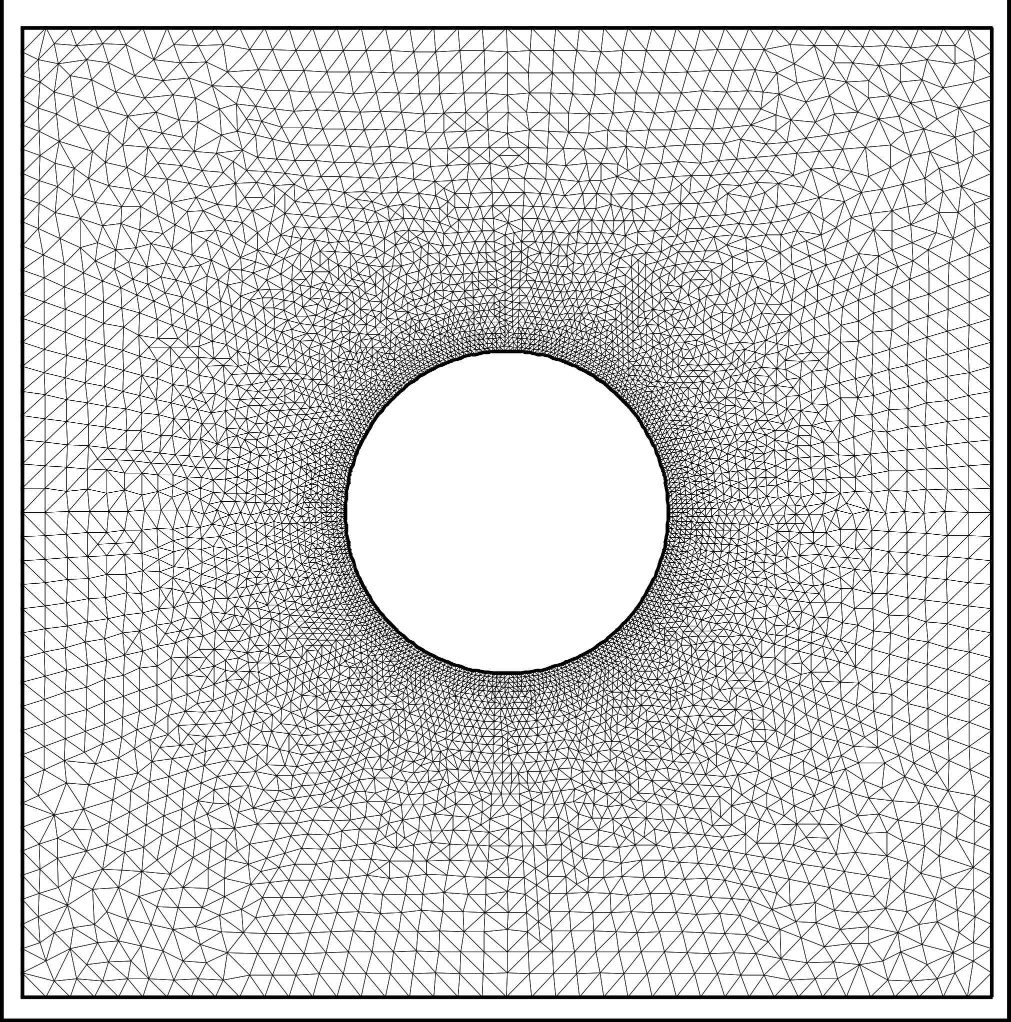
- **Solve the cell problem for** $\left(\widetilde{K}_{ij}, \widetilde{A}_j, \widetilde{W}\right)$
- **Solve the maro equations**

$$\boxed{\frac{\partial}{\partial X_k}\left[\left(n + M_{ik}\frac{\partial \phi}{\partial X_i}\right)\right] + (n+N)\frac{\partial^2 \phi}{\partial Z^2} = 0, \quad i,k = 1,2}$$

$$\begin{cases} \widetilde{p}^{(0)} = \mathrm{i}\phi, \quad \langle \widetilde{u}_i^{(0)} \rangle = n\dfrac{\partial \phi}{\partial X_i} + M_{ik}\dfrac{\partial \phi}{\partial X_k}, \quad \langle \widetilde{w}^{(0)} \rangle = (n+N)\dfrac{\partial \phi}{\partial Z} \\ M_{ik} = \displaystyle\oint_{S_B} ds \left[-\widetilde{A}_k \delta_{ij} + \sigma\left(\frac{\partial \widetilde{K}_{ik}}{\partial x_j} + \frac{\partial \widetilde{K}_{jk}}{\partial x_i}\right)\right] n_j \\ N = \sigma \displaystyle\oint_{S_B} ds \, \frac{\partial \widetilde{W}}{\partial x_j} n_j, \quad \boldsymbol{M_{ik}(\vec{X}), N(\vec{X}) = \text{functions of } \nu_e} \end{cases}$$

Numerical solutions (FDM); Analytical solutions?

Sample cell problem: FEM solutions



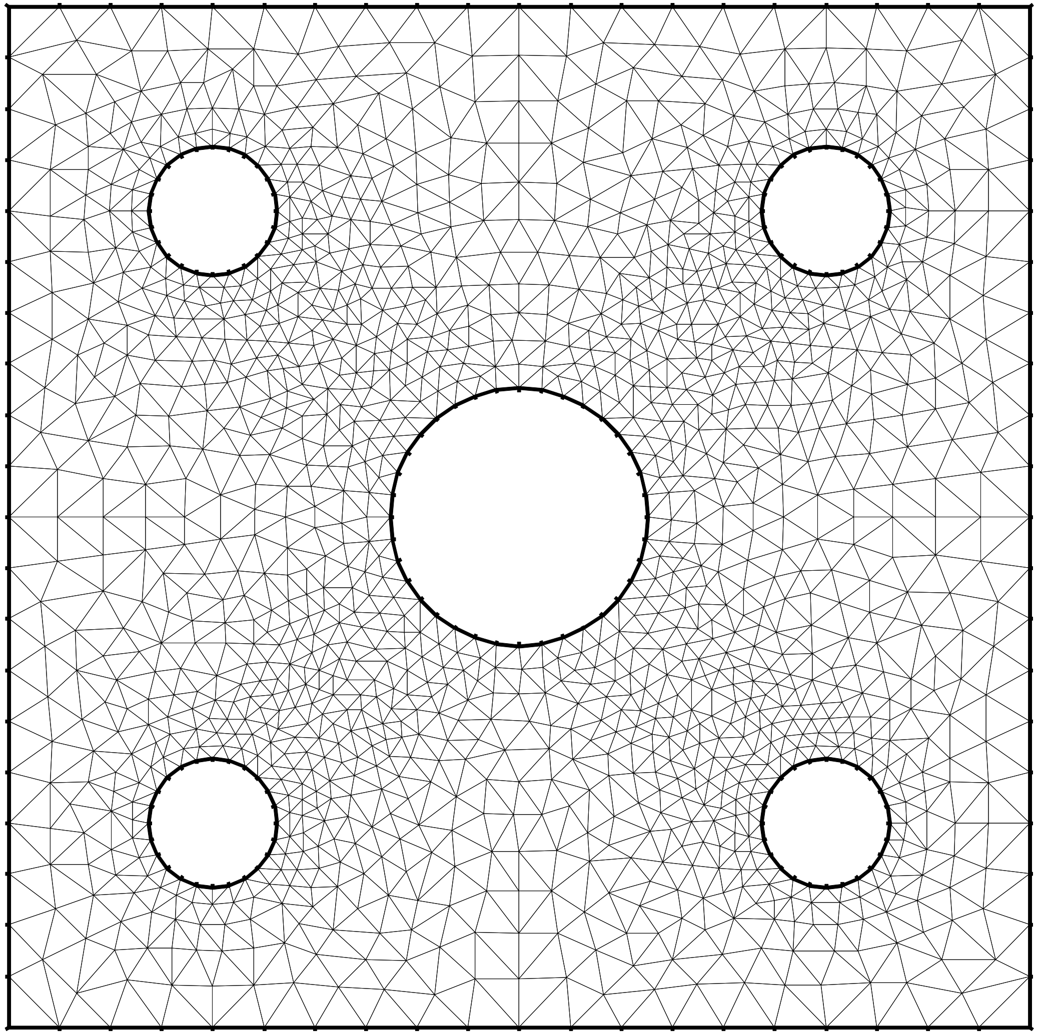
$$\begin{cases} \dfrac{\partial \widetilde{K}_{ij}}{\partial x_i} = 0 \\ -\,\mathrm{i}\widetilde{K}_{ij} = \delta_{ij} - \dfrac{\partial \widetilde{A}_j}{\partial x_i} + \sigma \dfrac{\partial^2 \widetilde{K}_{ij}}{\partial x_k \partial x_k} \end{cases}$$

$$-\,\mathrm{i}\widetilde{W} = 1 + \sigma \frac{\partial^2 \widetilde{W}}{\partial x_j \partial x_j}$$

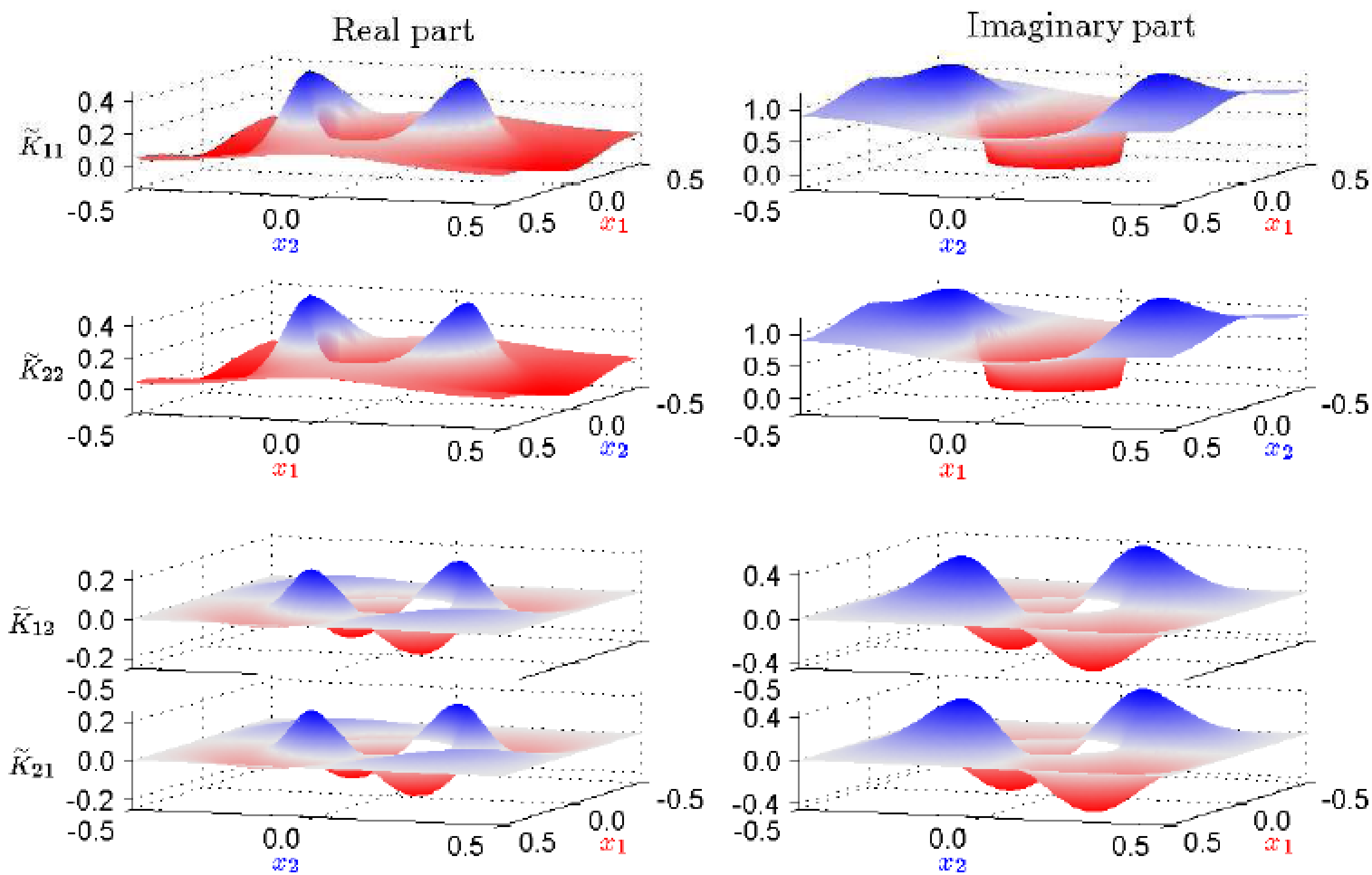
with

- **Periodicity**
- **No-slip on the cylinder** S_B
- $\langle \widetilde{A}_j \rangle = 0$

Several cylinders in a unit cell

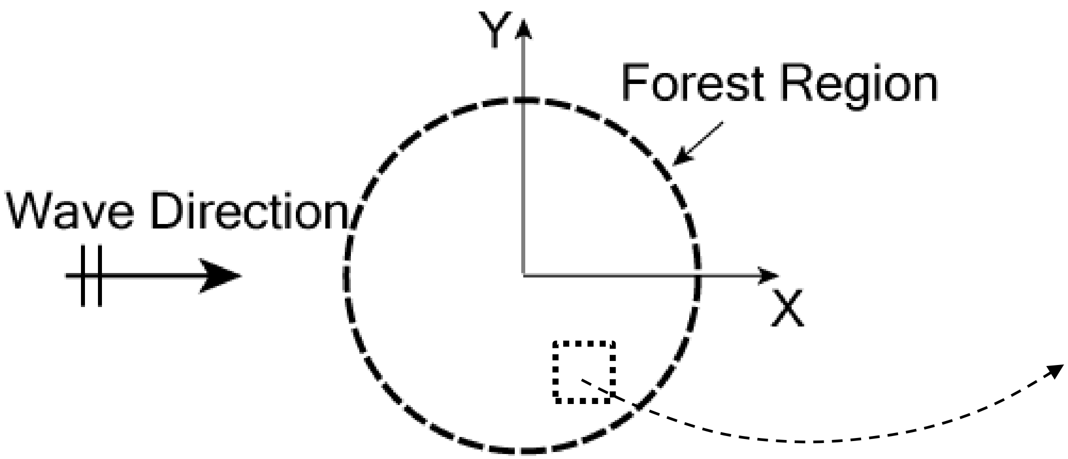


Sample cell solutions

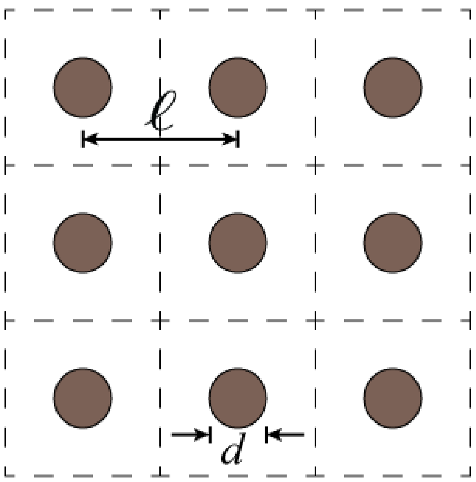


Macro-scale problem for constant eddy viscosity

Period wave through a circular forest



Sketch of the research problem



Arrangement of cylinder array

Leading-order governing equations
(dimensionless)

Complex potential

$$\tilde{p}^{(0)} = i\phi(X_i, Z)$$

- **Forest region:**

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \phi^{(\mathrm{F})}}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 \phi^{(\mathrm{F})}}{\partial \theta^2} + \left(\frac{n+N}{n+M}\right)\frac{\partial^2 \phi^{(\mathrm{F})}}{\partial z^2} = 0, \ r < R$$

- **Outside region:**

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \phi^{(\mathrm{I})}}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 \phi^{(\mathrm{I})}}{\partial \theta^2} + \frac{\partial^2 \phi^{(\mathrm{I})}}{\partial z^2} = 0, \ r > R$$

Complex coefficients:

$$M_{ik} = \frac{1}{\Omega}\oint_{S_B} ds\left[-\tilde{A}_k\delta_{ij} + \sigma\left(\frac{\partial \tilde{K}_{ik}}{\partial x_j} + \frac{\partial \tilde{K}_{jk}}{\partial x_i}\right)\right] n_j^{S_B}$$

$$M_{11} = M_{22} \equiv M \quad ; \quad M_{12} = M_{21} = 0$$

$$N = \frac{1}{\Omega}\sigma\oint_{S_B}\left(\frac{\partial \tilde{W}}{\partial x_j} n_j^{S_B}\right) ds$$

Constant for each cell

Macro-scale analytical solutions

- **Forest region:**

$$\phi^{(\mathrm{F})} = A_0 \sum_{m=0}^{\infty} \epsilon_m i^m \cos m\theta \left[\sum_{q=0}^{\infty} B_{mq} J_m(\gamma \hat{k}_q r) \frac{\cosh \hat{k}_q (Z+h)}{\cosh \hat{k}_q h} \right]$$

$$\gamma = \sqrt{(n+N)/(n+M)}$$

ϵ_m : Jacobi symbol

- **Outside region:**

Incident waves

$$\phi^{(\mathrm{I})} = A_0 \sum_{m=0}^{\infty} \epsilon_m i^m \cos m\theta \left\{ \left[J_m(k_0 r) + C_{m0} H_m^{(1)}(k_0 r) \right] \frac{\cosh k_0 (Z+h)}{\cosh k_0 h} \right\}$$

$$+ A_0 \sum_{m=0}^{\infty} \epsilon_m i^m \cos m\theta \left[\sum_{p=1}^{\infty} C_{mp} K_m(\kappa_p r) \frac{\cos \kappa_p (Z+h)}{\cos \kappa_p h} \right]$$

- **Matching conditions along** $r = R$

$$\phi^{(I)} = \phi^{(F)}, \quad -h < Z < 0$$

$$\frac{\partial \phi^{(I)}}{\partial r} = (n+M) \frac{\partial \phi^{(F)}}{\partial r}, \quad -h < Z < 0$$

Eddy viscosity by energy balance

- **Energy balance concept**
  - **Viscous dissipation = Work input**
- **Wave force on one or more cylinders**
  - **A single cylinder: Morison equation**

$$\text{Force} = \frac{\rho}{2} C_D u|u|d + \rho C_M \frac{\partial u}{\partial t} \frac{\pi d^2}{4}$$

  - **Multiple cylinders (idealized coastal forests)**
    - **No information on drag and inertia coefficients**
    - **Focus on time-averaged energy (work-dissipation) balance:**

$$\frac{1}{T}\int_t^{t+T} \mathcal{E} dt = \overline{\mathcal{E}} = \frac{1}{T}\int_t^{t+T} \Psi dt = \overline{\Psi}$$

Time-invariant eddy viscosity

- **Averaged rate of work done and dissipation**

$$\overline{\mathcal{E}} = \frac{1}{2}\rho d C_D \int_{-h}^{0} \overline{\langle u_1^{(0)} \rangle^2 \left| \langle u_1^{(0)} \rangle \right|} dZ + \mathcal{O}(\epsilon)$$

$$\overline{\Psi} = \rho \nu_e \frac{1}{\Omega} \int_{-h}^{0} dZ \iint_{\Omega_f} \overline{\frac{1}{2}\left(\frac{\partial u_i^{(0)}}{\partial x_j} + \frac{\partial u_j^{(0)}}{\partial x_i} \right)^2 + \left(\frac{\partial w^{(0)}}{\partial x_i} \right)^2} d\Omega$$

- **Energy balance**

$$\overline{\mathcal{E}} = \overline{\Psi} \quad \Rightarrow \quad \frac{\nu_e(X)}{\omega a_0 d} = \frac{\frac{2}{3\pi} C_D \left| \langle \widetilde{K}_{11} \rangle \right|^3 \int_{-h}^{0} \left| \frac{\partial \widetilde{p}^{(0)}}{\partial X} \right|^3 dZ}{\mathcal{F}_K \int_{-h}^{0} \left| \frac{\partial \widetilde{p}^{(0)}}{\partial X} \right|^2 dZ + \mathcal{F}_W \int_{-h}^{0} \left| \frac{\partial \widetilde{p}^{(0)}}{\partial Z} \right|^2 dZ},$$

$$\mathcal{F}_K = \mathcal{F}_K(\widetilde{K}_{ij}), \quad \mathcal{F}_W = \mathcal{F}_K(\widetilde{W})$$

Bulk (constant) eddy viscosity and drag coefficient

- **Averaged rate of work done and dissipation**

$$\overline{\Psi} = \rho\nu_e \frac{1}{\Omega} \iint_{\text{Forest}} dA_{\text{F}} \int_{-h}^{0} dZ \iint_{\Omega_f} \overline{\frac{1}{2}\left(\frac{\partial u_i^{(0)}}{\partial x_j} + \frac{\partial u_j^{(0)}}{\partial x_i}\right)^2 + \left(\frac{\partial w^{(0)}}{\partial x_i}\right)^2} d\Omega$$

$$\overline{\mathcal{E}} = \frac{1}{2}\rho d C_D \iint_{\text{Forest}} dA_{\text{F}} \int_{-h}^{0} \overline{\left\langle u_1^{(0)} \right\rangle^2 \left| \left\langle u_1^{(0)} \right\rangle \right|} dZ + \mathcal{O}(\epsilon)$$

- **Energy balance**

Bulk eddy viscosity

Bulk drag coefficient

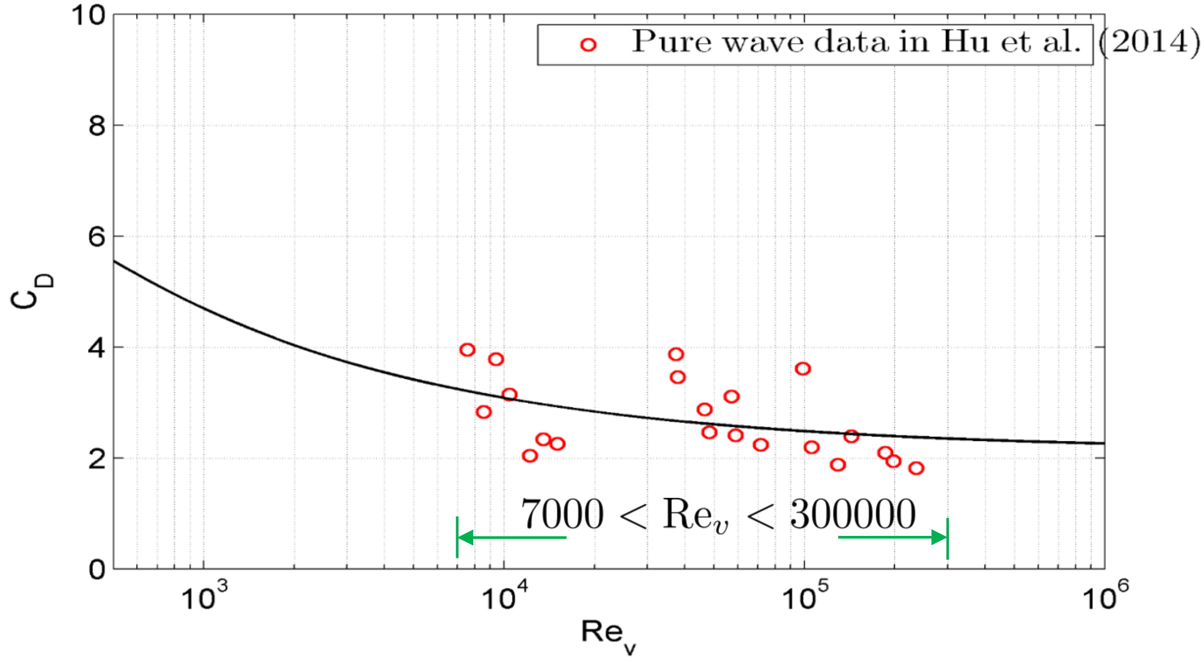
$$\frac{\langle \nu_e \rangle}{\omega a_0 d} = \frac{\frac{2}{3\pi} \langle C_D \rangle \iint_{\text{Forest}} dA_{\text{F}} \left| \langle \widetilde{K}_{11} \rangle \right|^3 \int_{-h}^{0} \left| \frac{\partial \widetilde{p}^{(0)}}{\partial X} \right|^3 dZ}{\iint_{\text{Forest}} \left[\mathcal{F}_K \int_{-h}^{0} \left| \frac{\partial \widetilde{p}^{(0)}}{\partial X} \right|^2 dZ + \mathcal{F}_W \int_{-h}^{0} \left| \frac{\partial \widetilde{p}^{(0)}}{\partial Z} \right|^2 dZ \right] dA_{\text{F}}}$$

Hu et al. (2014) for bulk drag coefficient:

- Conduct flume tests on a forest belt with finite thickness 6 m (normal incidence)
- Use rigid cylinders to model the forest
- Directly measure the forces on individual cylinders
- Use the same time-averaged formula to calculate the drag coefficient
- Use both submerged and emergent cylinders
- Propose a new drag coefficient formula for periodic waves and combined wave-current conditions

Bulk drag coefficient

Hu et al. experiments (2014)



$$\langle C_D \rangle = \frac{50}{\mathrm{Re}_v^{0.43}} + 2.13\left[1 - \exp\left(-\frac{\mathrm{Re}_v}{120.74}\right)\right]$$

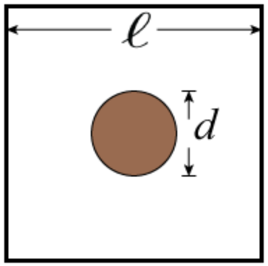
- **Reynolds number**

$$\mathrm{Re}_v = \frac{U_{\mathrm{mid}} r_v}{\nu}$$

- **Hydraulic radius**

$$r_v = \frac{\pi}{4} d \left(\frac{n}{1-n}\right)$$

- **Porosity**



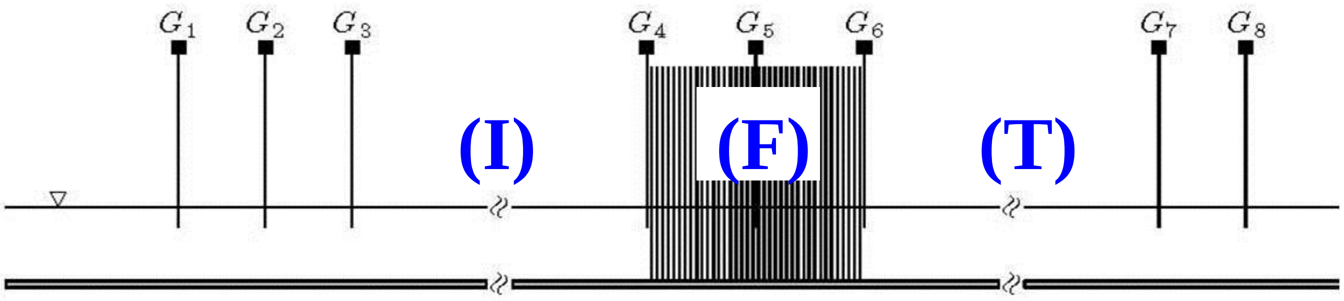
$$n = 1 - \frac{\pi d^2/4}{\ell^2}$$

Iterative scheme for eddy viscosity

- **Eddy viscosity is considered as a constant in the entire forest.**
- **Iteration for eddy viscosity**
  - **An initial guess for eddy viscosity**
  - **Solve the cell problem first**
  - **Solve the macro problem based on the cell solutions**
  - **Calculate the new eddy viscosity**
  - **Convergence test: around 10 iterations**

Model testing & application

Intermediate depth: a 2HD example



$$\phi^{(\mathrm{I})} = A_0 \frac{\cosh k_0(Z+h)}{\cosh k_0 h} e^{\mathrm{i}k_0 X} + A_0 R \frac{\cosh k_0(Z+h)}{\cosh k_0 h} e^{-\mathrm{i}k_0 X} + \sum_{p=1}^{\infty} A_p \frac{\cos \kappa_p(Z+h)}{\cos \kappa_p h} e^{\kappa_p X}$$

$$\phi^{(\mathrm{F})} = \sum_{q=0}^{\infty} \frac{\cosh \hat{k}_q(Z+h)}{\cosh \hat{k}_q h} \left[C_q e^{\mathrm{i}\gamma \hat{k}_q X} + D_q e^{-\mathrm{i}\gamma \hat{k}_q X} \right]$$

$$\phi^{(\mathrm{T})} = A_0 T \frac{\cosh k_0(Z+h)}{\cosh k_0 h} e^{\mathrm{i}k_0 X} + \sum_{p=1}^{\infty} B_p \frac{\cos \kappa_p(Z+h)}{\cos \kappa_p h} e^{-\kappa_p X}$$

$(R,\, T,\, A_p,\, B_p,\, C_q,\, D_q):$ Obtained by matching velocity & pressure along interfaces

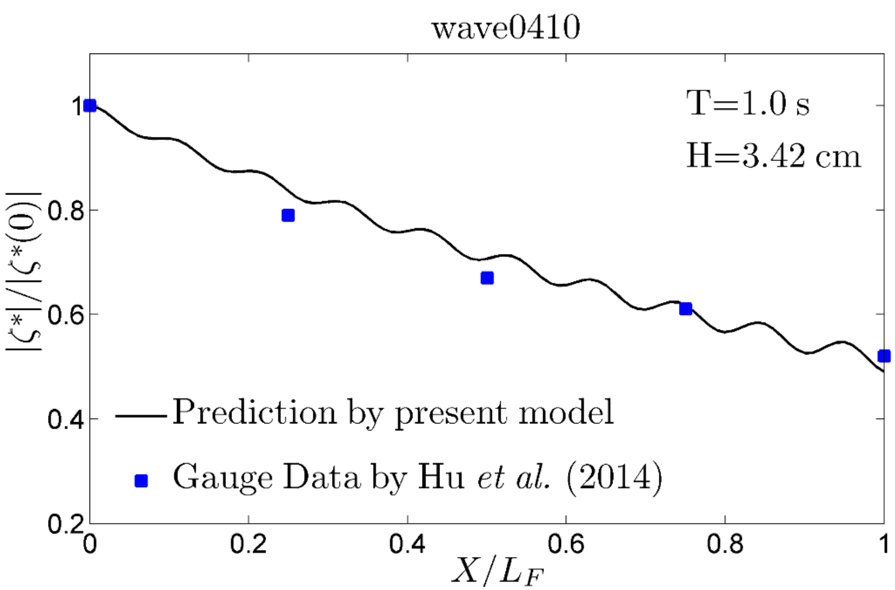
Dispersion relationships:

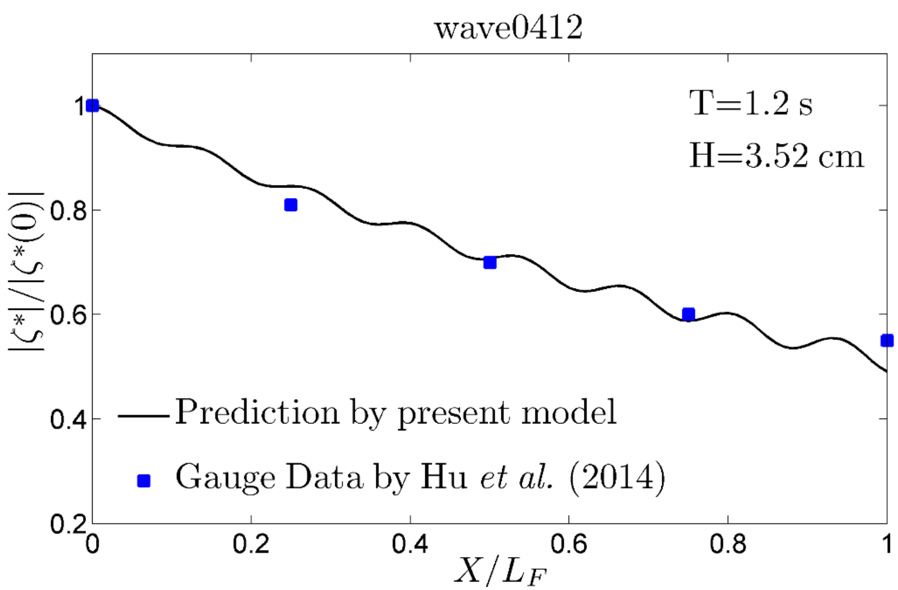
$$1 = k_0 \tanh k_0 h \;;\; 1 = -\kappa_p \tan \kappa_p h \; (p \geq 1)\,; \qquad 1 = \left(\frac{n+N}{n} \right) \hat{k}_q \tanh \hat{k}_q h, \quad (q \geq 0)$$

Model validation (Hu et al. 2014)

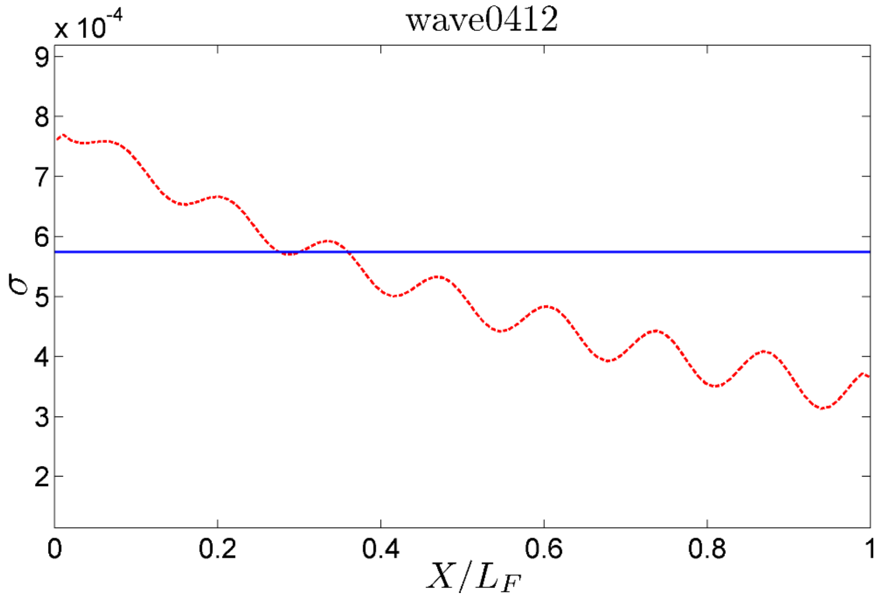
- **Two-dimensional forest belt with normal incidence**
  - ➢ **Compare the dimensionless relative wave amplitude inside the forest with the wave gauge measurements**

$$\frac{|\zeta^*|}{|\zeta^*(0)|}$$

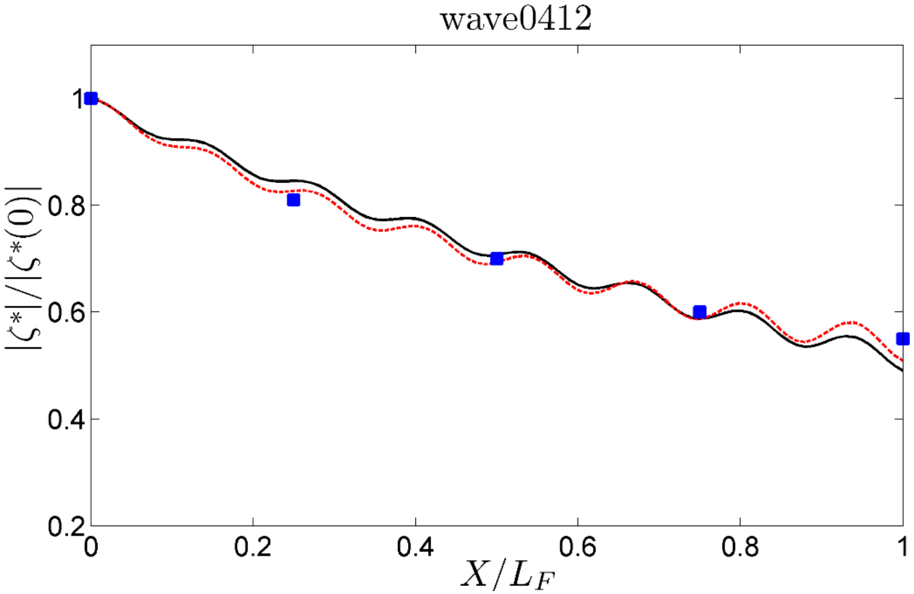




How appropriate is the constant eddy viscosity model?



Bulk eddy viscosity
vs.
Varying eddy viscosity



Normalized dimensionless wave amplitude

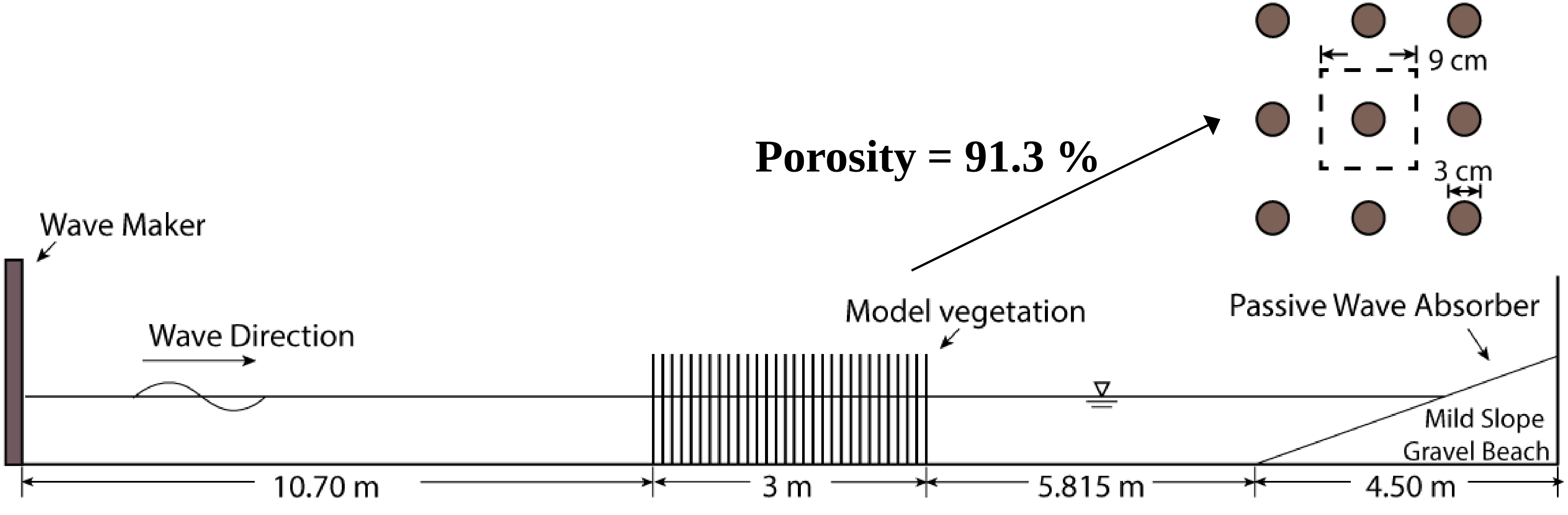
NEW

LABORATORY EXPERIMENTS

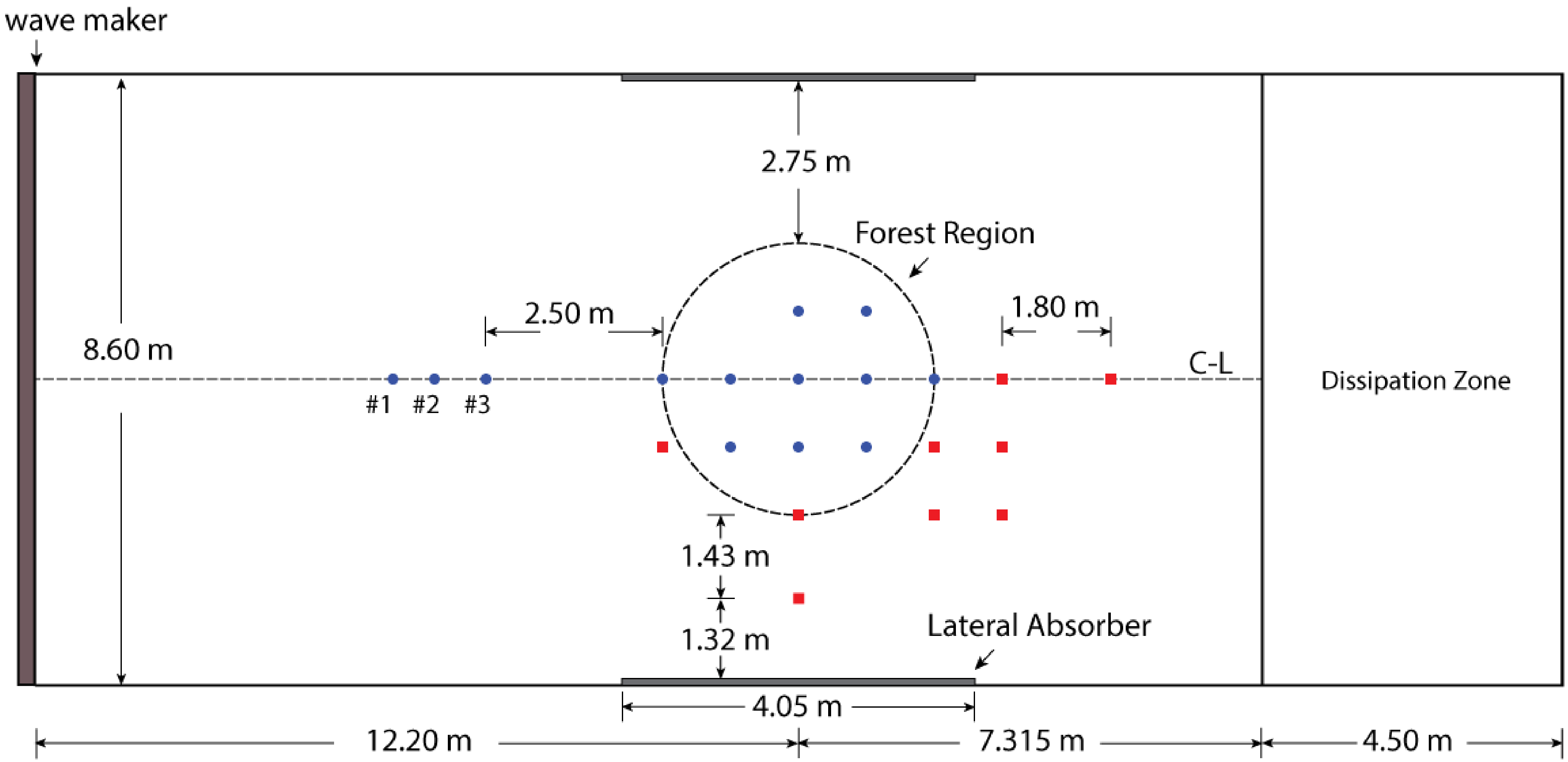
UNIVERSITY OF CANTABRIA, SPAIN

Experimental setup

- Wave basin: 28 m long, 8.6 m wide and 1.2 m deep
- Wave maker: piston type with 10 independent paddles
- Passive wave absorber: mild slope gravel beach (1:12)
- Forest region: circle with 3 m in diameter, 880 cylinders in use
- Circular cylinder: 50 cm long, 3 cm in diameter

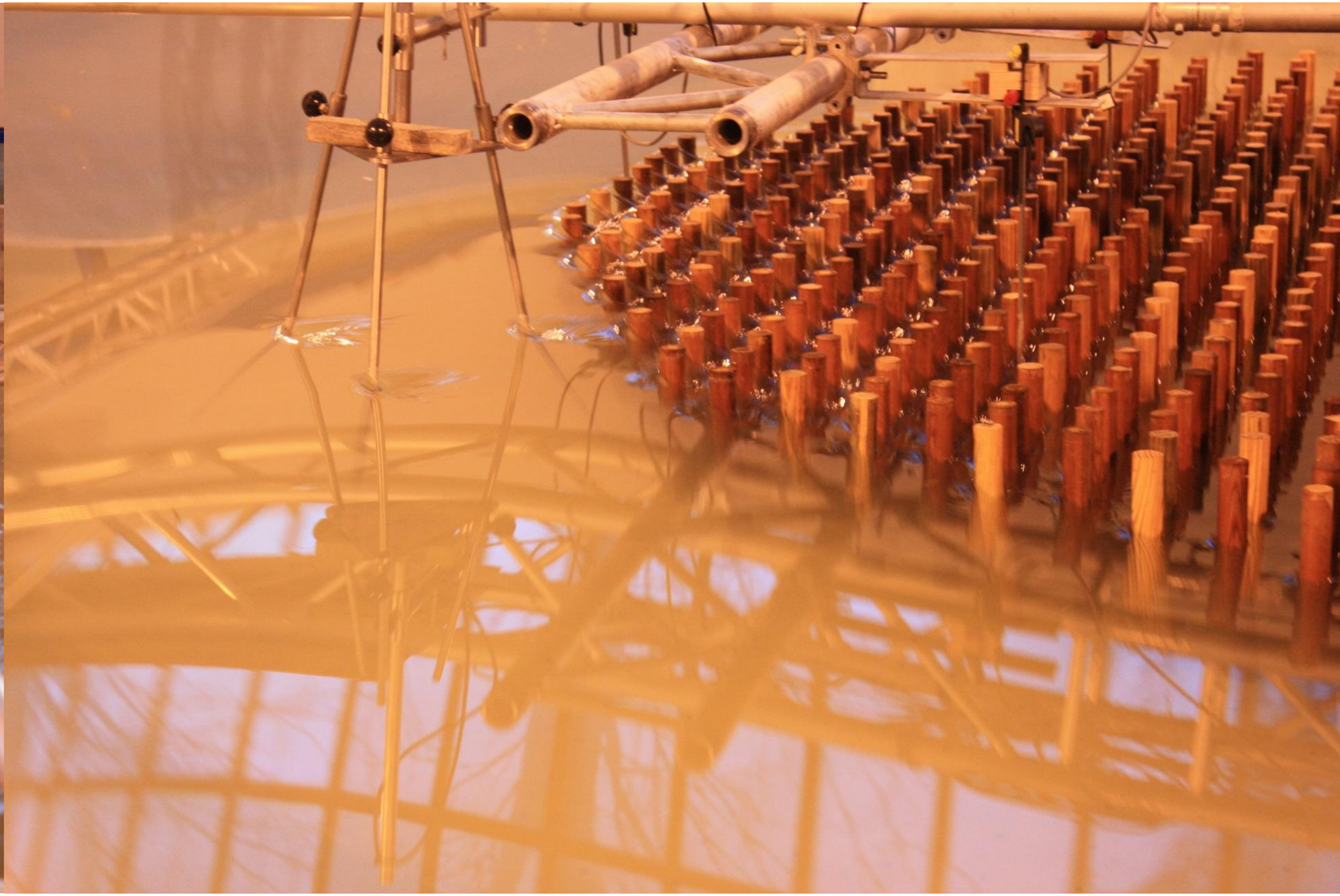


Experimental setup (cont.)



- **Instrumentation: 22 wave gauges**

Experiments



Experimental conditions

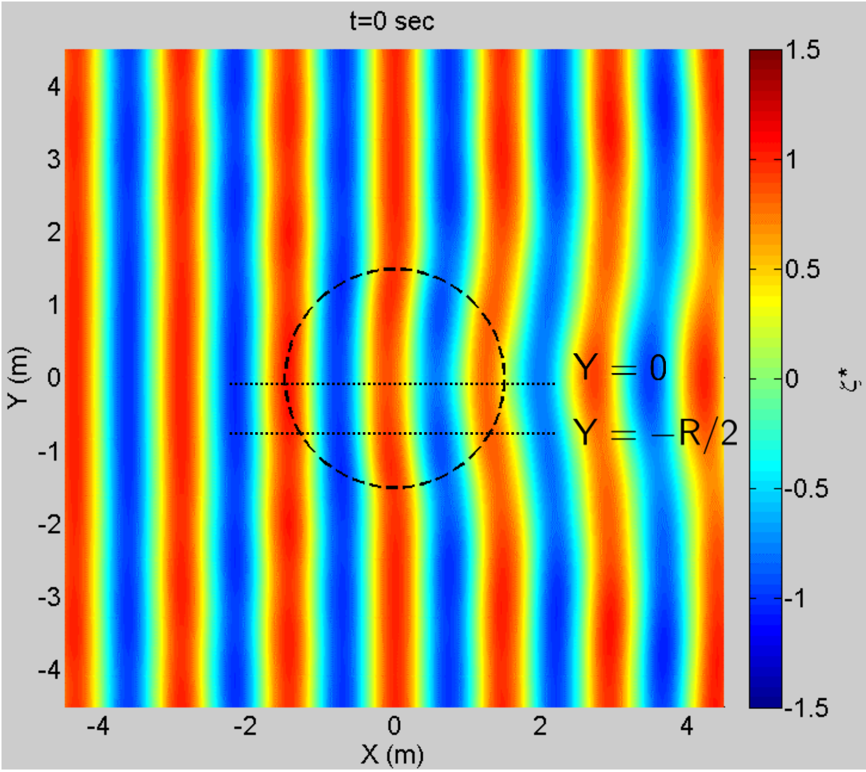
- **Two water depths were tested (h=30 cm, 40 cm)**
  - **Wave period ranges from 1.00 sec to 2.75 sec**
  - **Incident wave height ranges from 2.50 cm to 7.56 cm**

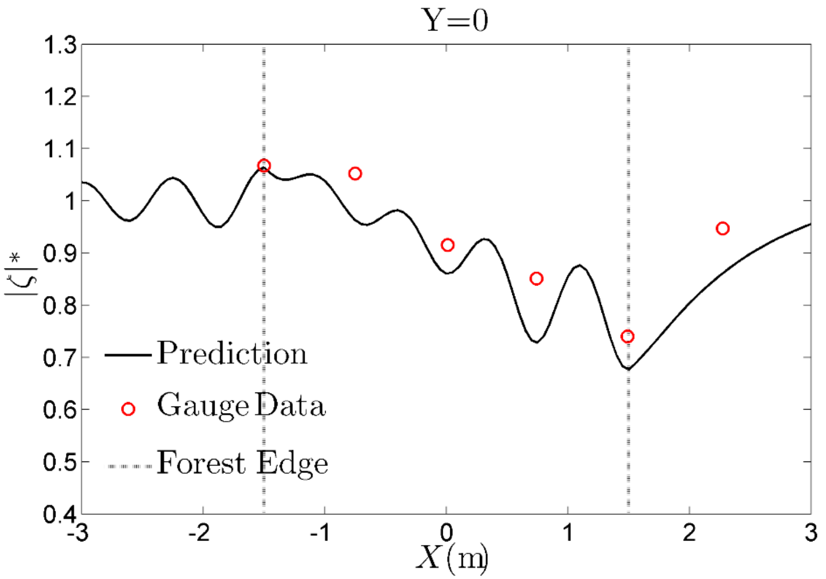
Table 1. Frequency Test II

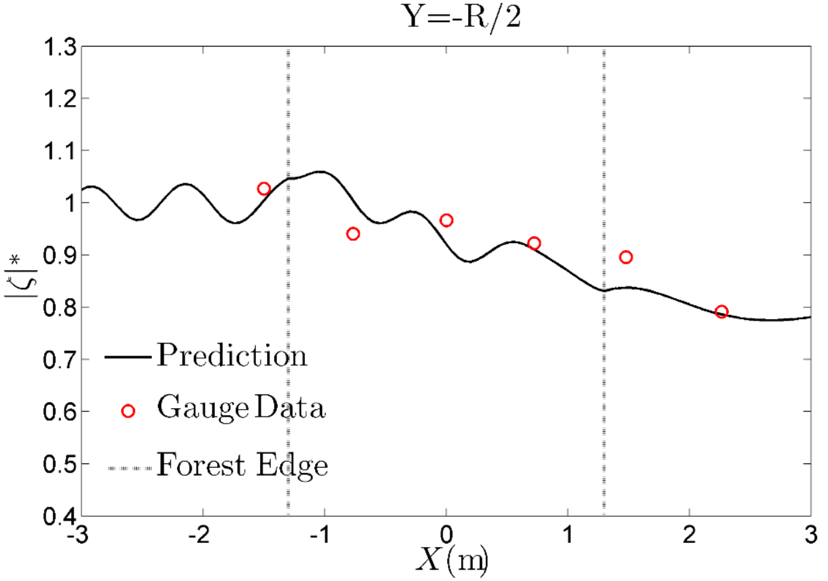
Case	h (cm)	T (s)	H (cm)	L (m)	kh	kA	Re_v	C_D	❆
4F2	40	1.00	4.90	1.464	1.717	0.105	9829	3.088	7.04E-04
4F3		1.25	5.54	2.052	1.225	0.085	13361	2.970	1.28E-03
4F4		1.50	5.38	2.616	0.961	0.065	14275	2.946	1.85E-03
4F5		1.75	5.30	3.162	0.795	0.053	14768	2.934	2.50E-03
4F6		2.00	5.22	3.695	0.680	0.044	15017	2.928	3.21E-03
4F7		2.25	5.04	4.220	0.596	0.038	14902	2.931	3.88E-03
4F8		2.50	5.06	4.739	0.530	0.034	15192	2.924	4.73E-03
4F9		2.75	5.22	5.254	0.478	0.031	15744	2.912	5.78E-03

Numerical results vs. data (a)

4F2	**Water depth**	**0.40 m**
	Wave period	**1.00 s**
	Wave length	**1.46 m**
	Wave height	**4.90 cm**

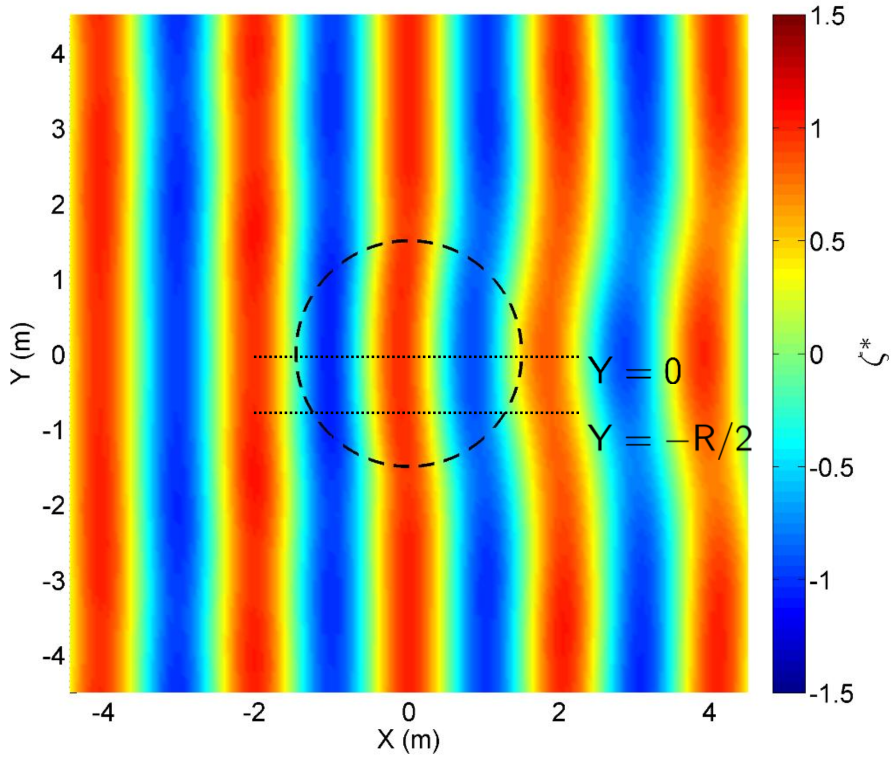


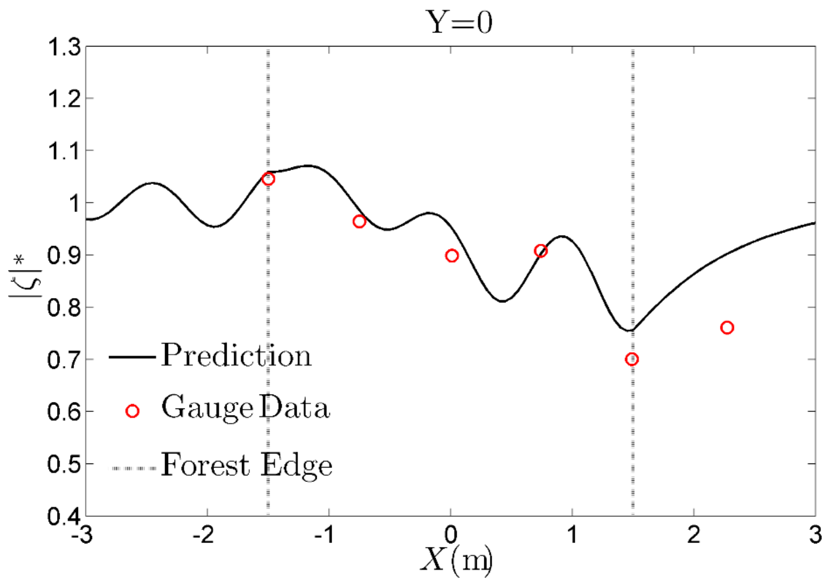


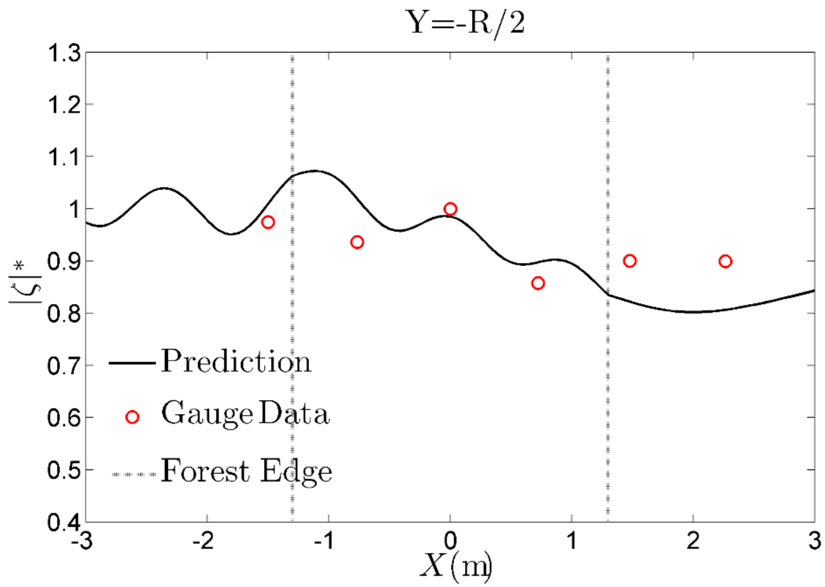


Numerical results vs. data (b)

4F3	**Water depth**	**0.40 m**
	Wave period	**1.25 s**
	Wave length	**2.05 m**
	Wave height	**5.54 cm**

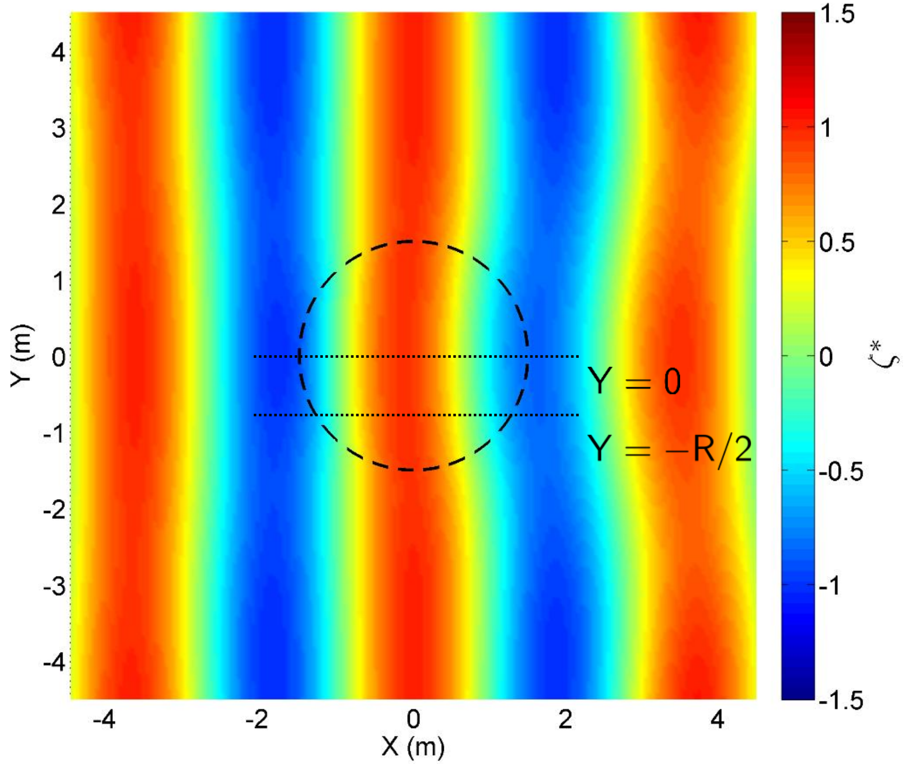


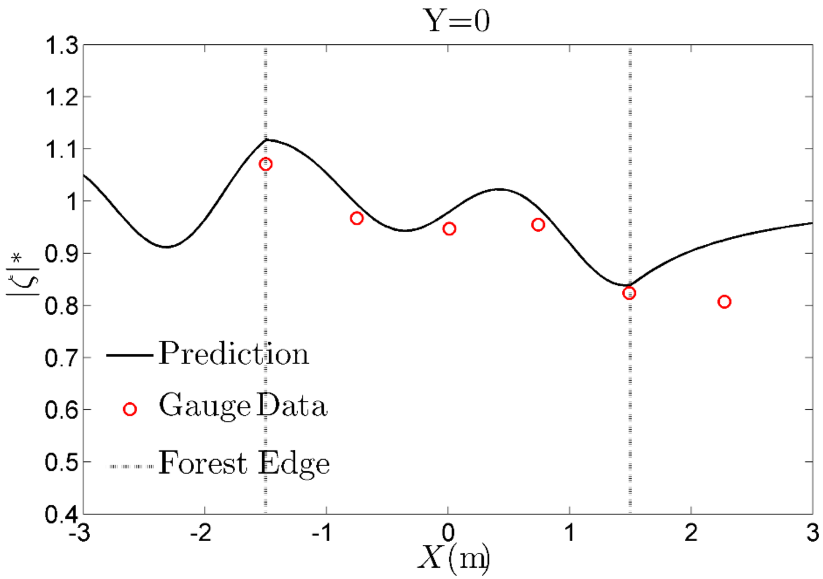


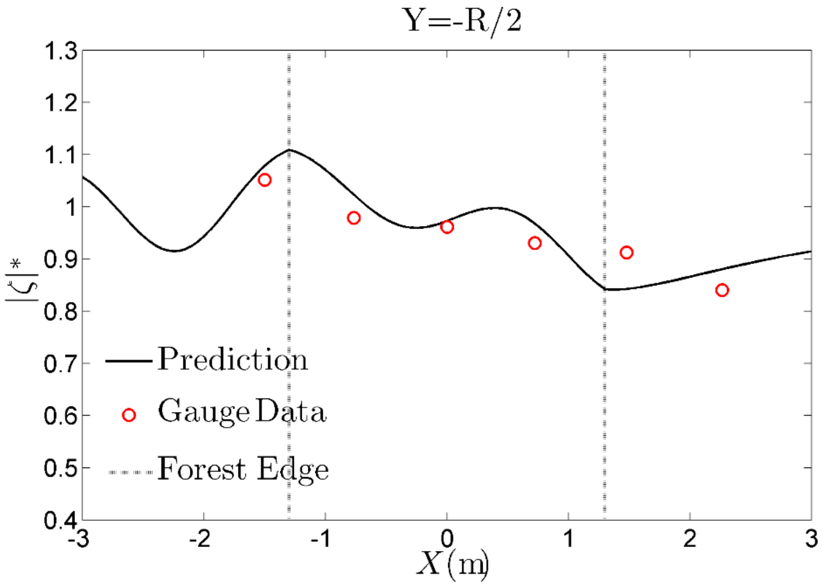


Numerical results vs. data (c)

4F6	**Water depth**	**0.40 m**
	Wave period	**2.00 s**
	Wave length	**3.70 m**
	Wave height	**5.22 cm**







Surface waves through emergent coastal trees

Summary

- A macro-theory for wave motions, with effective coefficients obtained numerically from the micro-scale problem
- Analytical and numerical solutions have been discussed
- New eddy viscosity model
- Good agreements between the theory and the experimental data

Important facts

- Strong wave attenuation
- Considerable reflected waves
- Theory can be used as a design guideline

Improvements

- Choice of eddy viscosity: need more experimental works!
- Weakly wave nonlinearity

End.

Thanks!

Questions?